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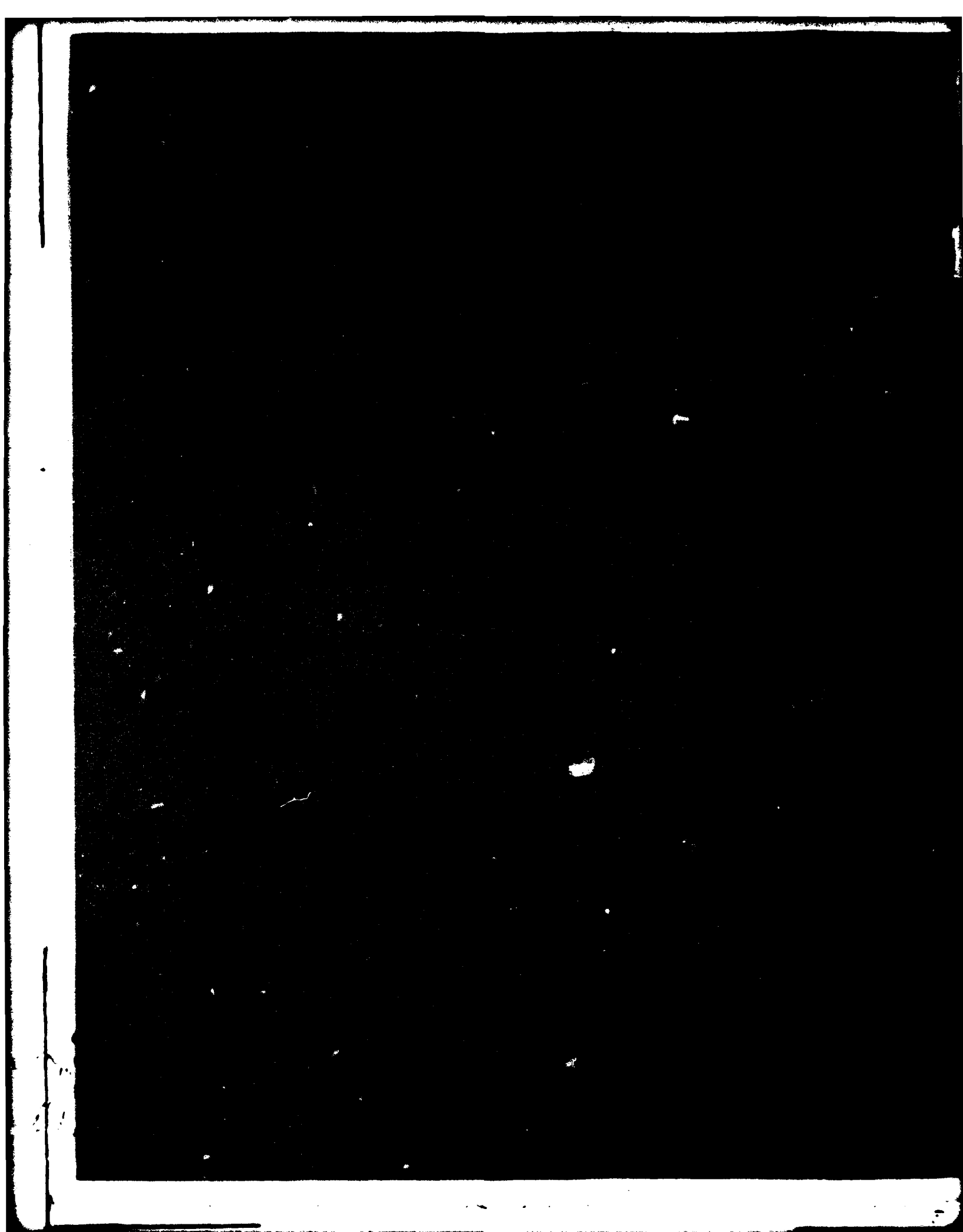
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Trajectories of particles emitted normally from a charged sphere into a constant magnetic field are examined. Calculations are made which separate such trajectories into two classes: those that return to the sphere and those that do not.

Preface

I wish to thank Pat Bench for programming the numerical solutions.

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Emission of Particles From a
Charged Sphere Into a Magnetic Field

1. INTRODUCTION

A number of experiments[1-3] dealing with the emission of beams of charged
particles from material bodies into the earth's magnetosphere have been reported
in the literature. Since many of the calculations relating to these experiments
attempt to anticipate and to more or less model actual ambient conditions, they
are so involved that little insight into the basic mechanisms that influence the
emission process is gained. To fill this gap, a prototype problem — namely, to
calculate in the presence of a spherically symmetric electric field, the effect of
an assumed constant magnetic field on particles emitted normally from a sphere —
has been solved. In particular, if β is the ratio of cyclotron to sphere radius and
θ the angle between the direction of emission and that of the magnetic field, rela-
tionships between β and θ which separate all emitted particles into two classes —
those that return to the sphere and those that do not — are obtained. The result of
this exercise is a set of curves, one for each sphere potential, that give insight
into the role of the magnetic field in general emissive processes.

1. Hendrickson, R.A., McEntire, R.W., and Winkler, J.R. (1971) Nature 230:
    564-566.

2. Cambou, F., et al (1978) Nature 271:723-726.

3. Hess, W. N. (1969) Science 164:1512-1513.

Calculations of actual charge densities and potential distributions surrounding the sphere require self-consistent solutions of charged particle trajectory and Poisson equations, a lengthy and difficult procedure. Here we do not carry out these calculations, but rather obtain qualitative indications of the effect of space charge by assuming potentials proportional to $1/r'^N$ (r' = radial coordinate) and determining separation curves for each of several integral values of N. Although not all potentials, $\phi(r')$, can be represented by a uniformly convergent power series, $\sum\limits_{n=0}^{\infty} \dfrac{a_n}{r'^n}$, in the interval $r_0 \leq r' \leq \infty$, one can, by summing terms, obtain information on a fairly large class of potentials.

2. CALCULATIONS

If we assume a potential of the form

$$\phi = \phi_0 \left(\frac{r_0}{r'}\right)^N \quad , \tag{1}$$

the appropriate equations and initial conditions for this problem are the following:

$$m \frac{d^2 \vec{r'}}{dt^2} = e \left(\frac{d\vec{r'}}{dt} \times \vec{B} - N \frac{\vec{r'} r_0^N \phi_0}{r'^{N+2}} \right)$$

$$\vec{r'} = \vec{j} r_0 \sin \theta + \vec{k} r_0 \cos \theta \tag{2}$$

$$\frac{d\vec{r'}}{dt} = \vec{j} v_0 \sin \theta + \vec{k} v_0 \cos \theta$$

$$t = 0 \quad . \tag{3}$$

Here

  m, e = particle mass and charge respectively

  $\vec{r'}$ = radius vector

  r' = $|\vec{r'}|$

  $\vec{B}$ = magnetic field vector

  r_0 = sphere radius

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ϕ_o = sphere potential

$\vec{j}, \vec{k}$ = unit vectors along rectangular y and z axes respectively

θ = angle between magnetic field and emission direction

v_o = initial velocity

t = time .

By the following changes in variables,

$$\vec{r} = \vec{r}'/r_o$$

$$\tau = \frac{eB_o}{m} t$$

$$\beta = \frac{m v_o}{r_o e B_o}$$

$$\alpha = \frac{m \phi_o}{r_o^2 e B_o^2}$$

$(B_o = |\vec{B}|)$, Eq. (2) may be put into the rectangular component, non-dimensional form:

$$\frac{d^2 x}{d\tau^2} = \frac{dy}{d\tau} - \alpha N \frac{x}{r^{N+2}}$$

$$\frac{d^2 y}{d\tau^2} = - \frac{dx}{d\tau} - \alpha N \frac{y}{r^{N+2}} \tag{4}$$

$$\frac{d^2 z}{d\tau^2} = - \alpha N \frac{z}{r^{N+2}}$$

with

$x = 0$ $\qquad\qquad$ $y = \sin \theta$ $\qquad\qquad$ $z = \cos \theta$

$\frac{dx}{d\tau} = 0$ $\qquad\qquad$ $\frac{dy}{d\tau} = \beta \sin \theta$ $\qquad\qquad$ $\frac{dz}{d\tau} = \beta \cos \theta$

$\tau = 0$

as corresponding initial conditions.

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If these equations are converted to an equivalent set of six first-order equations, they may be solved by use of standard computer codes. To generate the curves that separate escaping from returning trajectories, values of α and β (subject to the energy constraint $\beta \geq \sqrt{2\alpha}$) are chosen. Starting with $\theta \cong \pi/2$, which guarantees a return, trajectories are taken for values of θ successively decreased by 0.05 radian until an escape is obtained. Starting with the value of θ at which an escape is obtained, trajectories are taken for values of θ successively increased by 0.005 radian until a return is obtained. This process is continued until the desired accuracy of the value of θ is obtained.

3. RESULTS

The results for N = 1 are shown in Figure 1. For any pair of values (θ, β) falling above a given curve, the particle escapes; for any pair falling below a given curve, the particle returns to the sphere. With no magnetic field, the separation curves would be continuations of the straight horizontal lines out to $\theta = \pi/2$; these lines are representations of the dimensionless energy relationship $\alpha = \beta^2/2$. The portions of the curves that deviate from the straight lines thus give the effects of the magnetic field in returning particles to the sphere.

For $\alpha = 0$, the trajectory is readily solved for analytically. We have, for this case,

$$r^2/r_o^2 = 1 + 2\,\beta^2\,\sin^2\theta + 2\,\beta\,\sin^2\theta\,\sin\tau - 2\,\beta^2\,\sin^2\theta\,\cos\tau$$

$$+ \beta^2\,\tau^2\,\cos^2\theta + 2\,\beta\,\tau\,\cos^2\theta \quad .$$

The condition that particles be recollected is that the equation $r^2/r_o^2 = 1$ have a solution for some $\tau > 0$. The relationship between β and θ that separates escaping from recollected particles will then be given by eliminating τ from the simultaneous implicit equations:

$$F = 2\tan^2\theta\,[\beta(1 - \cos\tau) + \sin\tau] + \beta\tau^2 + 2\tau = 0 \tag{5}$$

$$\frac{1}{2}\frac{dF}{d\tau} = \tan^2\theta\,[\beta\sin\tau + \cos\tau] + \beta\tau + 1 = 0 \quad , \tag{6}$$

where

$$F = (r^2/r_o^2 - 1)\,\frac{1}{\cos^2\theta} \quad .$$

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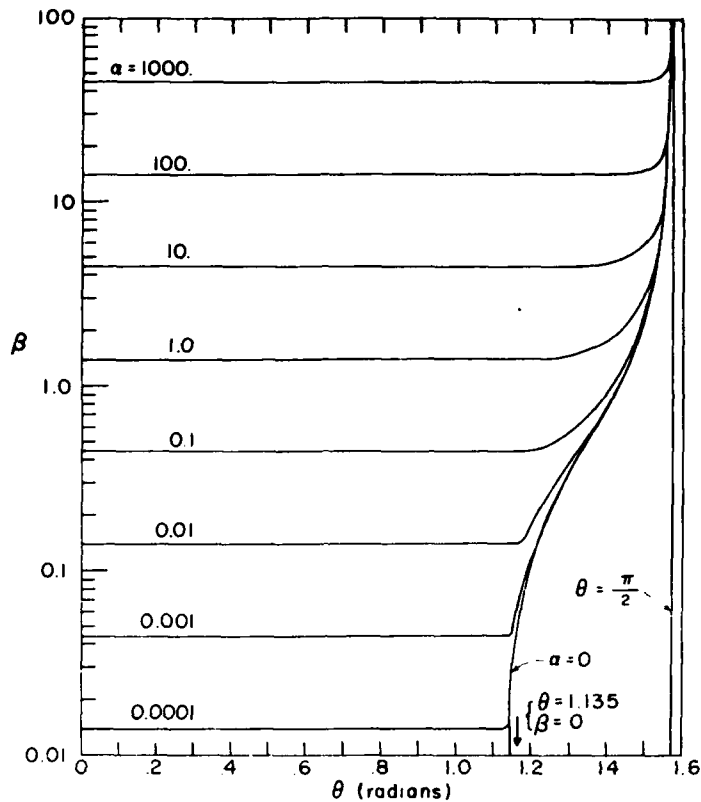Figure 1. β, Ratio of Cyclotron to Sphere Radius, vs θ, Angle Between the Direction of Emission and That of the Magnetic Field, for Various Values of α and $N = 1$. The curves separate all emitted particles into those which return to the sphere (below curve) and those which do not (above curve)

For large values of β, Eqs. (5) and (6) can be solved analytically to obtain

$$\tan \theta = 2\pi\beta \quad , \tag{7}$$

which, for large β, agrees well with numerical solutions of Eqs. (5) and (6) and also with solutions of Eqs. (4) with $\alpha = 0$. It is seen from Figure 1 that for $\alpha = 0$ there is an angle $\theta \cong 1.13$ radians below which all particles escape independent of initial energy.

The simulated effects of space charge are demonstrated in Figures 2, 3, and 4 where, for values of $\alpha = 0.01$, 1.0, and 100, separation curves for various values of N are plotted. Also included on each figure is a "thin sheath limit."
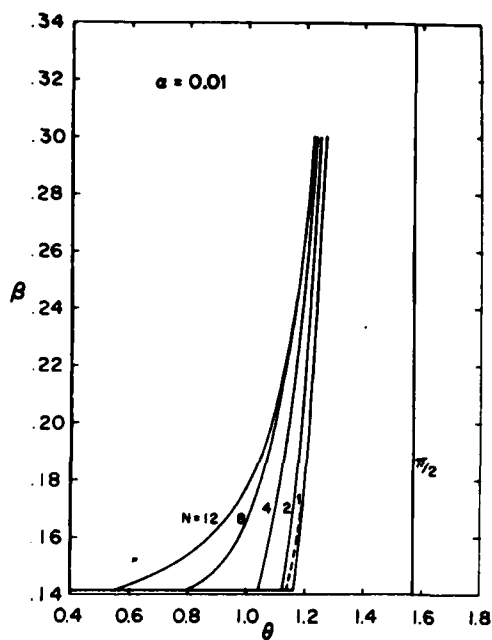
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Figure 2. Separation Curves for α = 0.01 and Various Values of N. The dashed curve is the thin sheath limit
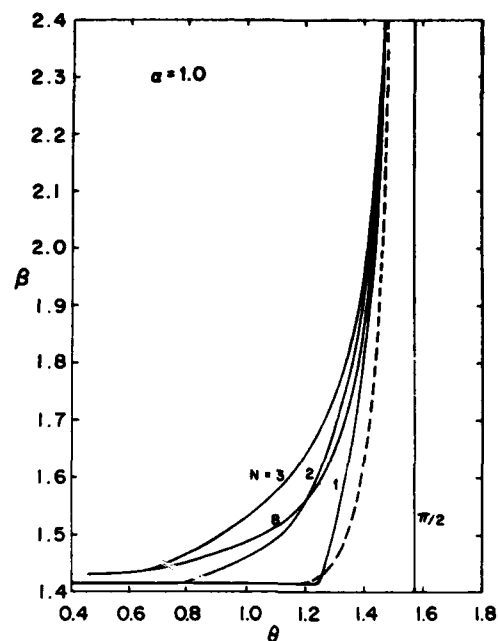

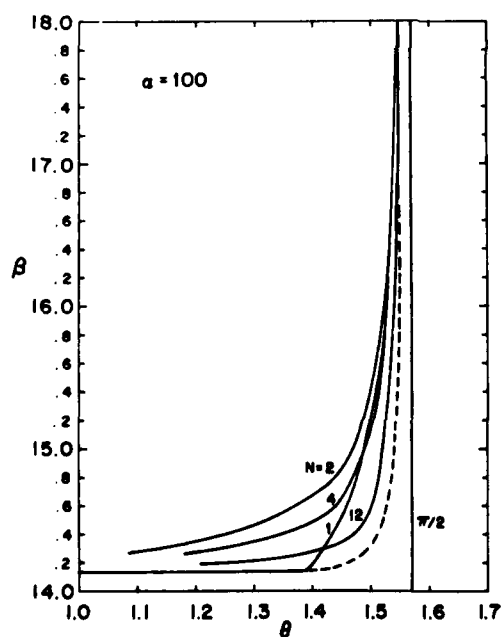Figure 3. Separation Curves for α = 1.0 and Various Values of N. The dashed curve is the thin sheath limit

Figure 4. Separation Curves for α = 100 and Various Values of N. The dashed curve is the thin sheath limit

This is obtained by plotting θ, as taken from the $\alpha = 0$ separation curve, as a function of $\beta' = \sqrt{\beta^2 - 2\alpha}$. The reasoning is that in this limit the particle loses energy, α, in traversing an infinitesimally thin sheath and then moves in a field-free region starting from r = 1, with an initial velocity characteristic of the reduced energy.

One might expect that: (A) separation curves for successively higher powers of N would form a monotonic sequence, and (B) the thin sheath limit would be approached as N → ∞. In fact, inspection of the figures shows that (A) is definitely not true while (B) may or may not be. For $\alpha = 100$, the curve that deviates most from the thin sheath limit (returns most particles to the sphere) is that for N = 2, while for $\alpha = 1.0$, the curve for N = 3 plays this role. Separation curves for $\alpha = 0.1$ show a maximum deviation from the thin sheath limit for N = 7. For $\alpha = 0.01$, the separation curves for values of N up to 12 form a monotonic sequence, but one that is diverging away from rather than converging to the thin sheath limit. It is possible that, as for the cases with larger α, there will be a reversal, and that for some higher N the curves will start to approach the thin sheath limit; but this is not known.

In general, changes in the shape of the separation curves corresponding to changes in the form of the potential seem to exhibit some complexity, with curves crossing each other and even, on occasion, crossing the thin sheath limit. Yet, in spite of this, a general statement can be made: unless angles of emission are high, particles having energies above the sphere potential will have a good chance of escaping. For example, if $\beta^2 = 4\alpha$ (initial kinetic energy = twice the sphere potential) and N = 12, for $\alpha = 0.01$ all particles having $\theta < 1.05$ radians escape, for $\alpha = 1.0$ all particles having $\theta < 1.40$ radians escape, and for $\alpha = 100$, particles having $\theta < 1.55$ radians escape.